

Composition of Reflections

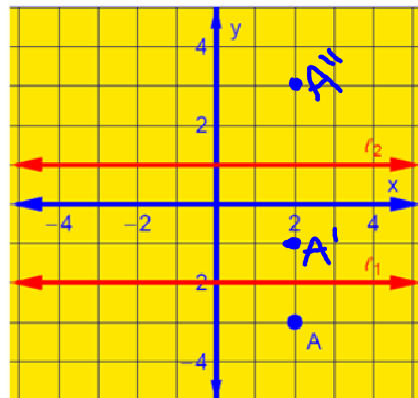
December 8, 2009

Objectives

- In an earlier section we defined a composition of transformations as any two transformations in which the second transformation is performed on the image of the first transformation.
- In this section we are going to investigate compositions of reflections in parallel and intersecting lines.
- We will see that reflections are related to the other isometries.

An example

- Reflect the point $A(2, -3)$ in l_1 and then reflect its image in l_2 .
- What do you notice about the distance between the original point and the final image point?
- Is there a single transformation that gets us from the first point to the last point?



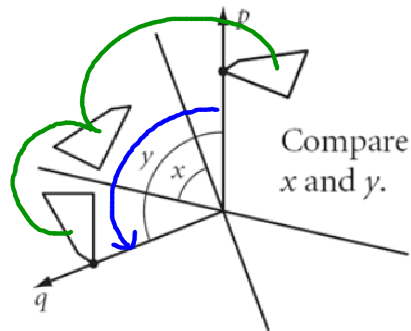
$\langle 0, 6 \rangle$ $(x+0, y+6)$
 reflection across $5x$

What we have learned so far

- A composition of reflections in two parallel lines is a translation.
- The translation glides all points through twice the distance between the lines.

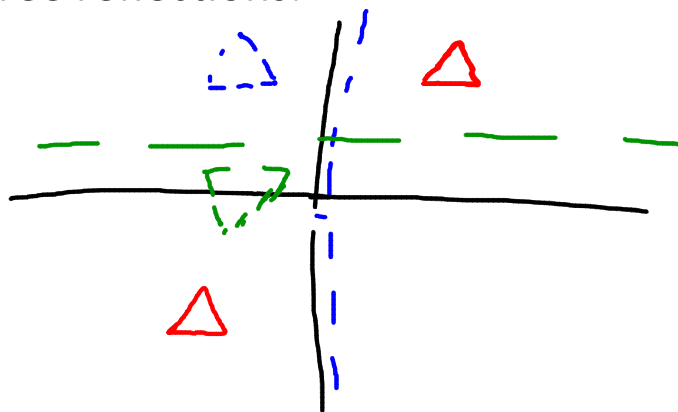
Intersecting Lines

- A composition of reflections in two intersecting lines is a rotation.
- The measure of the angle of rotation is twice the measure of the angle from the first line of reflection to the second.



More about composition of reflections

- In a plane, two congruent figures can be mapped onto one another by a composition of at most three reflections.



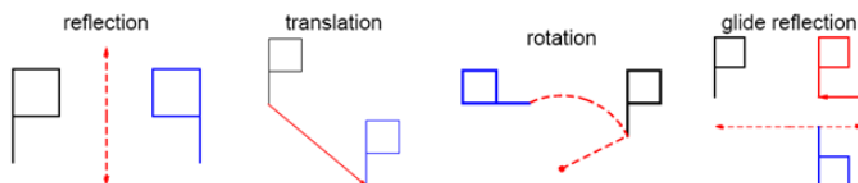
What is a glide reflection?

- A composition of three reflections in lines that intersect in more than one point is called a glide reflection.
- It is called a glide reflection because any such composition can be accomplished by gliding (or translating) the figure followed by a reflection in the a parallel to the translation vector.

Isometries

- You can map any two congruent figures onto one another by a single reflection, translation, rotation, or glide reflection. These four transformations are the only isometries.

Check out the four isometries



Using composition

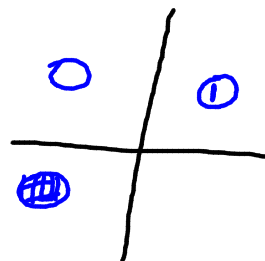
The notation for composition is \circ .

$S \circ T$ mean do T , then S .

$$S \circ T$$

$$R_x \circ R_y$$

\uparrow 2nd \uparrow 1st



For example

$$S:(x, y) = (x - 1, y + 2) \text{ and}$$

$$T:(x, y) = (x + 2, y - 1).$$

Draw S \circ T.
 $(x+1, y+1)$

